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ANALYTICAL EXPRESSIONS FOR THE PARAMETERS OF FINNED AND RIDGED WAVEGUIDES

Wolfgang J.R. Hoefer and Miles N. Burton,
Department of Electrical Engineering,
University of Ottawa,
Ottawa, Ontario, Canada K1N 6N5

Abstract

Novel closed-form expressions for the cutoff frequency and the characteristic impedance of finned and ridged waveguides are presented. Agreement with previously published numerical data is better than one percent for all parameters of practical interest. The expressions considerably facilitate computer-aided design and tolerance analysis of ridged waveguide structures without compromise in accuracy.

Introduction

Ridged waveguides find many applications by virtue of their large inherent bandwidth and low characteristic impedance. Furthermore, planar microwave and millimeter wave circuits of the type described by Konishi [1], [2] as well as fin lines [3], [4] can be analyzed and designed using ridged waveguide theory.

The ridged waveguide is well documented [5] - [9] by tabulated results or design diagrams and graphs which are necessarily restricted to a few cross-sectional dimensions. If a different geometry is needed, the designer must either solve a transcendental equation (Transverse Resonance Method) or use a numerical technique.

In this paper, original closed-form expressions for the cutoff frequency and the characteristic impedance of the dominant mode in double-and single-ridged waveguide are derived. The special case of waveguides with thin ridges (finned waveguides) is treated first. Then, the more general case of ridges with finite thickness is considered. Expressions are based on perturbation theory and contain empirical correction terms to assure agreement better than $\pm$ one percent with various numerical techniques.

Analysis of Double-Ridged Waveguide Using Perturbation Theory

Cutoff Frequency

Ridges in the E-plane of a waveguide lower the cutoff frequency of the dominant mode through capacitive loading. If the ridges are short and thin, their effect can be accurately evaluated through perturbation theory.

Waveguides with Short Fins. Fig. 1(a) shows a region bounded by parallel conducting strips on top and bottom, and by magnetic walls on each side. The electrostatic field is perturbed by a thin conducting band suspended in the center. According to Wheeler [11], the relative increase in static capacitance of the line due to the band can be expressed as a ratio of effective areas:

$$\Delta C_0/C_0 = A_e/A \qquad (d' \ll b,a) \qquad (1)$$

where $A_e = d'^2\pi/4$ is the effective area (circumscribed circle) of the band. $A = ab$ is the cross-section of the line, and C_0 is its static capacitance before introduction of the band. The same expression applies

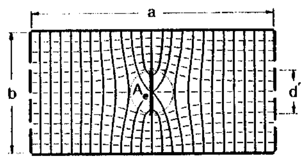

Fig. 1(a) A conducting band increases the capacitance of a parallel-plate waveguide.

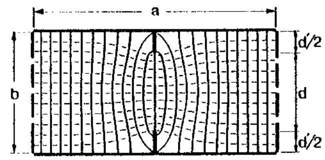

Fig. 1(b) Reciprocal structure presenting the same increase in waveguide capacitance.

to the reciprocal structure shown in Fig. 1(b) which is similar to a ridged waveguide.

To obtain a ridged waveguide the magnetic walls are replaced by electric walls, thus imposing a sinusoidal transverse field distribution. At cutoff, the equivalent line capacitance is proportional to the stored field energy and is half the static value: $C_1 = C_0/2$.

The variation ΔC_1 introduced by the fins is virtually equal to ΔC_0 since the field is quasi-uniform in the centre. Thus, for short fins ($b-d \ll b,a$):

$$\Delta C_1/C_1 = 2\Delta C_0/C_0 = (b-d)^2\pi/(2ab) \qquad (2)$$

Since the magnetic field is practically unperturbed by the fins, the shift in cutoff frequency is solely due to the change in capacitance:

$$f_{c0}/f_{cr} = \lambda_{cr}/\lambda_{c0} \doteq [1+(\Delta C_1/C_1)]^{1/2} \qquad (3)$$

where $f_{c0} = c/\lambda_{c0}$ is the cutoff frequency of the unperturbed waveguide, and $f_{cr} = c/\lambda_{cr}$ is the cutoff frequency of the ridged waveguide.

With $\lambda_{c0} = 2a$, the normalized cutoff frequency of the waveguide with short fins ($b-d \ll b$) becomes:

$$b/\lambda_{cr} \doteq (0.5\ b/a)\ [1+(0.5\pi b/a)(1-d/b)^2]^{-1/2} \qquad (4)$$

Waveguides with Long Fins. Eq. (4) cannot be applied to waveguides with long fins ($d/b \ll 1$), which are of greater practical interest. Fortunately, from Marcuvitz's work on the susceptance of capacitive windows [8], a relation between the capacitances of complementary long and short fins can be derived. Fig. 2 presents two such complementary structures. Neglecting higher order terms in eqs. (2b) and (2c) of sec. 5.1 in [8], and setting $d'_a = d_b$, the ratio of the fin capacitances in both cases is obtained as:

$$\begin{aligned}\Delta C_b/\Delta C_a &= [(B_b/Y_0)(\lambda_{cb}/b)]/[(B_a/Y_0)(\lambda_{ca}/b)] \\ &= 2\ [2b/(\pi d_b)]^2 \ell n\ [2b/(\pi d_b)]\end{aligned} \qquad (5)$$

where the subscripts a and b pertain to short and long

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fins, respectively. The guided wavelength λ_g in [8] has been replaced by the cutoff wavelengths λ_{ca} and λ_{cb}.

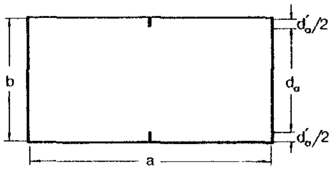

Fig. 2(a) Waveguide with short fins.

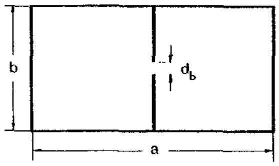

Fig. 2(b) Complementary waveguide with long fins.

Consequently, the normalized cutoff frequency of the guide with long fins ($d_b/b \ll 1$) is approximately:

$$b/\lambda_{cb} \doteq (0.5\ b/a)[1+(\Delta C_a/C_1)(\Delta C_b/\Delta C_a)]^{-1/2}$$
$$= (0.5\ b/a)[1+(4/\pi)(b/a)\ \ell n\ (2/\pi)(b/d_b)]^{-1/2} \qquad (6)$$

Waveguides with Fins of Any Length. Both eqs. (4) and (6) can be derived from one single expression (7):

$$b/\lambda_{cr} \doteq (0.5\ b/a)[1+(4/\pi)(b/a)\ell n\ csc(0.5\pi d/b)]^{-1/2} \qquad (7)$$

which transforms into (4) for $(b-d)/b \ll 1$, and into (6) for $d/b \ll 1$.

Since these expressions have been derived by assuming that the waveguide fields are only altered in the immediate vicinity of the fins, the accuracy of (7) deteriorates with increasing ratio b/a. It is therefore necessary to correct (7) by taking second order effects into account. The required correction cannot be determined analytically. However, it has been found empirically that if the second term in (7) is multiplied by a factor $1+0.2\sqrt{b/a}$, the resulting expression (8) agrees with various numerical methods [10], [13] to within one percent in the ranges $0 < b/a \leq 1$ and $0.01 \leq d/b \leq 1$.

$$b/\lambda_{cr} = (0.5b/a)[1+(4/\pi)(1+0.2\sqrt{b/a})(b/a)\ell n\ csc(0.5\pi d/b)]^{-\frac{1}{2}} \qquad (8)$$

Since the numerical techniques differ among themselves within this margin, the corrected perturbation formula (8) is equally reliable and accurate, and certainly more flexible than graphical design data.

Waveguides with Thick Ridges of Any Length. Ridges of finite thickness add a second capacitance ΔC_2 to the waveguide. To a first approximation, ΔC_2 is the capacitance of parallel plates of width s and separation d (see Fig. 3).

$$\Delta C_2 \doteq \varepsilon_0\ s/d \qquad (9)$$

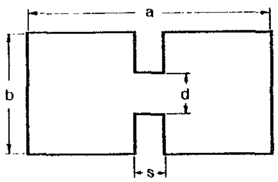

Fig. 3 Double Ridged Waveguide

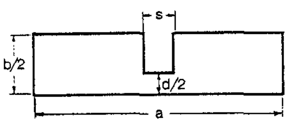

Fig. 4 Single-Ridged Waveguide

At the same time, the width of the "unperturbed" part of the waveguide is reduced from a to a-s, and its equivalent capacitance is reduced to

$$C_2 = 0.5\varepsilon_0(a-s)/b \qquad (10)$$

where the factor 0.5 stems from the sinusoidal distribution of the field in the guide. The relative change in capacitance is thus:

$$\Delta C_2/C_2 \doteq 2\ sb/[d(a-s)] \qquad (11)$$

By adding this term in the perturbation formula (8) for thin ridges, and after replacing a with a-s, an expression for the normalized cutoff frequency in guides with thick ridges is obtained. Again, second order effects can be included by multiplying (11) with an empirical correction term to fit numerical methods. The following expression is thus obtained for the normalized cutoff frequency of double-ridged waveguide:

$$b/\lambda_{cr} = \frac{b}{2(a-s)}[1+\frac{4}{\pi}(1+0.2\sqrt{\frac{b}{a-s}})\ \frac{b}{a-s}\ \ell n\ csc(\frac{\pi}{2}\ \frac{d}{b})$$
$$+\ (2.45+0.2\ \frac{s}{a})\ \frac{sb}{d(a-s)}]^{-1/2} \qquad (12)$$

This expression is equivalent to (7) for $s = 0$.

Finally, the guided wavelength for any frequency is related to the cutoff wavelength by

$$\lambda_g/\lambda = [1-(\lambda/\lambda_{cr})^2]^{-1/2} \qquad (13)$$

where λ is the free-space wavelength.

Characteristic Impedance

The characteristic impedance of ridged waveguide is not uniquely defined, but, whatever definition may be adopted, its value depends on the frequency as follows:

$$Z_0 = Z_{0\infty}[1-(\lambda/\lambda_{cr})^2]^{-1/2} \qquad (14)$$

where $Z_{0\infty}$ is the characteristic impedance for infinite frequency, and λ_{cr} is given by (12). Sharma and Hoefer [12] have derived the following formula for $Z_{0\infty}$:

$$Z_{0\infty} = \frac{120\pi^2(b/\lambda_{cr})}{\frac{b}{d}\ \sin\pi\ \frac{s}{b}\ \frac{b}{\lambda_{cr}} + [\frac{B_0}{Y_0} + \tan\ \frac{\pi}{2}\ \frac{b}{\lambda_{cr}}\ (\frac{a-s}{b})]\cos\pi\frac{s}{b}\ \frac{b}{\lambda_{cr}}} \qquad (15)$$

where a, b, s and d are defined in Fig. 3. The normalized susceptance B_0/Y_0 is approximated (Marcuvitz [8]) as:

$$B_0/Y_0 \doteq (2b/\lambda_{cr})\ell n\ csc\ \frac{\pi d}{2b} \qquad (16)$$

$Z_{0\infty}$ in (15) is a voltage-to-current ratio. The voltage is the integral of the electric field taken along a straight line joining the ridges in the middle of the guide. The current is the integral of the longitudinal surface current flowing in the top wall including the upper ridge.

Application to Single-Ridged Waveguide

All formulae derived above for double-ridged waveguide can be applied to the single-ridged waveguide with the following interpretation:

In the expression for the cutoff frequency (12), b is twice the height of the single-ridged guide, and d is twice the spacing between the ridge and the bottom wall (see Fig. 4).

The same interpretation applies to the expression for characteristic impedance (15). Finally, this impedance must be divided by two to obtain the value for single-ridged waveguide.

Discussion and Conclusion

The closed-form expressions for the cutoff frequency agree with numerical methods and previously published results [6], [8], [9] to within one percent in the following ranges: $0.01 \leq d/b \leq 1$, $0 < b/a \leq 1$ and $0 \leq s/a \leq 0.45$. This excellent agreement is evident from Figs. 5 and 6. Fig. 5 shows the normalized cutoff frequency of finned waveguide (8) against the background of values obtained with numerical methods.

In Fig. 6, the formula for the waveguide with ridges of finite thickness (12) is compared with data published by Hopfer [6]. Actually, Fig. 6 shows the normalized cutoff wavelength λ_{cr}/a which is related to the normalized cutoff frequency b/λ_{cr} as follows:

$$\lambda_{cr}/a = (b/a)/(b/\lambda_{cr}) \tag{17}$$

For $s/a > 0.45$, which are seldom of practical interest, the error increases beyond one percent because of the mounting effect of the side walls.

The accuracy of the expression for the characteristic impedance (15) is the same as that of (12) but its usefulness depends on the adequacy of its definition in a particular situation.

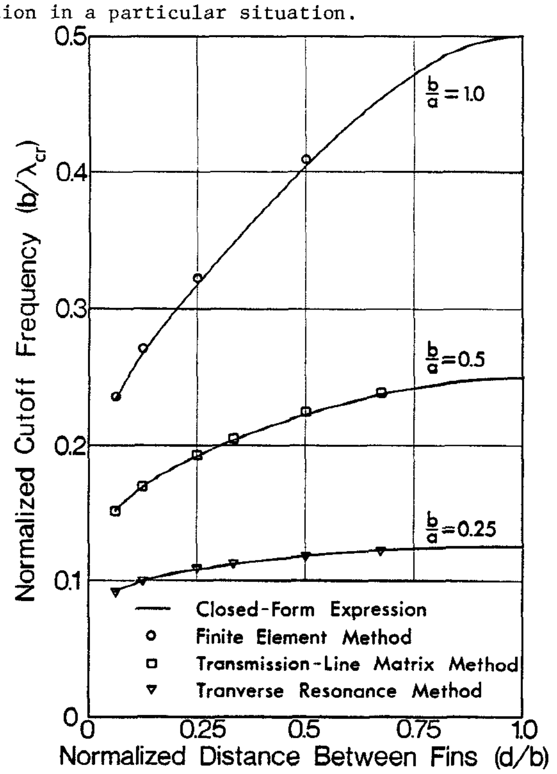

Fig. 5 Normalized cutoff frequency of finned waveguide ($s = 0$). The closed-form expression (8) is compared with numerical methods.

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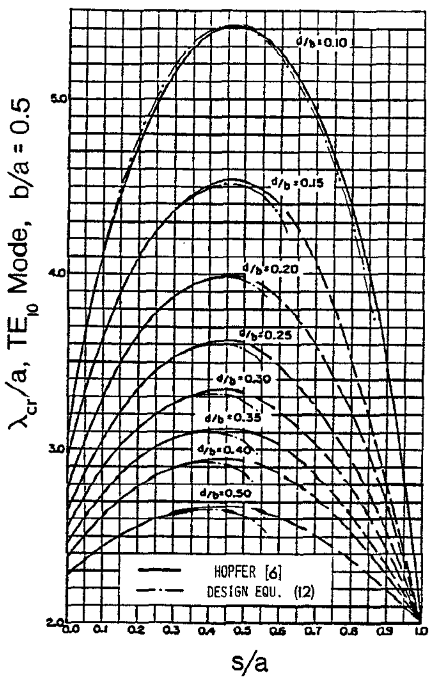

Fig. 6 Normalized cutoff wavelength of ridged waveguide. The closed-form expression (12) is compared with values published by Hopfer [6].

In conclusion, the empirically corrected perturbation formulae for ridged waveguide are as accurate as various numerical methods for all geometries of practical interest.

Because of their simplicity, these new expressions considerably simplify the design of ridged waveguides without any concession in accuracy. They have the advantage of great flexibility, can be differentiated directly for tolerance analysis, and may be easily programmed for computer aided design and manufacturing.

References

[1] Y. Konishi et al, IEEE MTT-22, pp. 869-73, Oct. 74

[2] Y. Konishi, IEEE MTT-26, pp. 716-19, Oct. 78

[3] P.J. Meier, IEEE MTT-22, pp. 1209-16, Dec. 74

[4] W.J.R. Hoefer, 1978 IEEE MTT-Symp. Dig. p.471

[5] S.B. Cohn, Proc. IRE, vol. 35, pp.783-88, Aug. 47

[6] S. Hopfer, IRE MTT-3, pp. 20-29, Oct. 55

[7] T.-S. Chen, IRE MTT-5, pp. 12-17, Jan. 57

[8] N. Marcuvitz, "Waveguide Handbook", Boston T.P. 64

[9] W.J. Getsinger, IRE MTT-10, pp.41-50, Jan. 62

[10] A.K.Sharma et al, 1981 IEEE AP-Symp.Dig. p.308,L.A.

[11] H.A. Wheeler, IEEE MTT-12, pp. 231-44, Mar. 64

[12] A.K. Sharma et al, IEEE MTT-30 to appear

[13] Y.C. Shih et al, IEEE MTT-28, pp. 743-46, Jul. 80